**8.Hamiltonian Mechanics and Time Translation Invariance**

**8.1 Time Translation Symmetry**

The Symmetry that relates to energy conservation includes a shift of time.

If we conduct an experiment from t0 to t1 and the results of that experiment match the results of an experiment that is the same as the first one, with the only difference being that it is conducted in a later time, namely ( t0 + Δt to t1+Δt), then we say that the system is invariant under time-translation .

Generally:

**If the shift doesn’t affect the outcome, we say that it is invariant over *time-translation.***

The Langrangian might vary with time, but **ONLY** because coordinates and velocities vary.

**Explicit time dependence:** The form of the Langrangian depends on time.

**A system is time-translation invariant if there is no explicit time dependence in its Langrangian.**

**8.2 Energy Conservation and The Hamiltonian**

Let’s try to see what happens when the value of the Langrangian changes as the system evolves.

Now we can examine each term and replace them so that

And

so combined these give

And now if we use the verse of the product rule we:

Now we define the Hamiltonian as

Or in other words H=E and

Now in a system where PE = V(q) The Langrangian is

The Momentum is So since

So we see that with simple algebra we get

Or

If a system is time-translation invariant, H or Energy is conserved.

Energy = Hamiltonian

**Hamilton’s Equations:**

**So by knowing all the values of the coordinates and momenta at any time and the form of the Hamiltonian, we can determine a trajectory throughout phase-space.**

**8.3 Derivation of Hamilton’s Equations**